Linear relations

Recall the equation for a straight line is $y = mx + b$

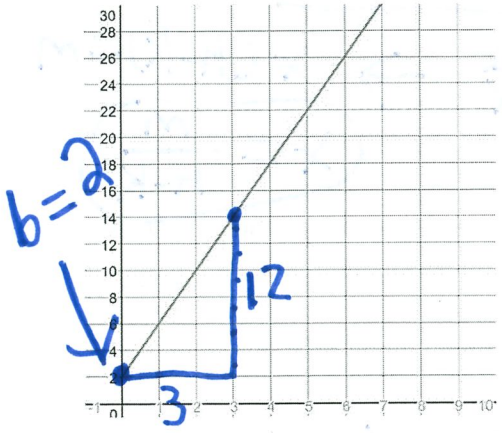
Where x is the independant variable

y is the dependant variable

m is the slope of the line

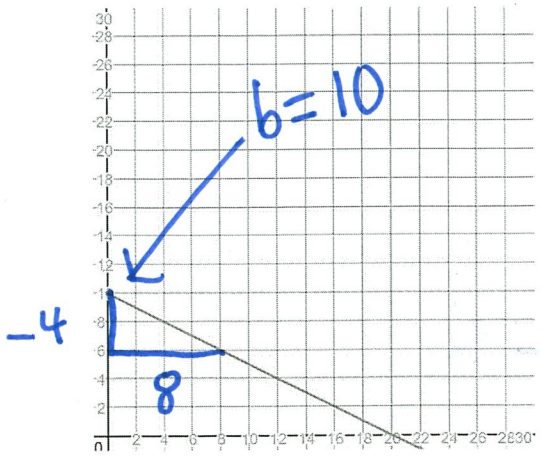
This equation tells you how y changes as x changes

m = slope = $\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_f - y_i}{x_f - x_i}$



$m = \frac{12}{3} = 4$

$y = 4x + 2$



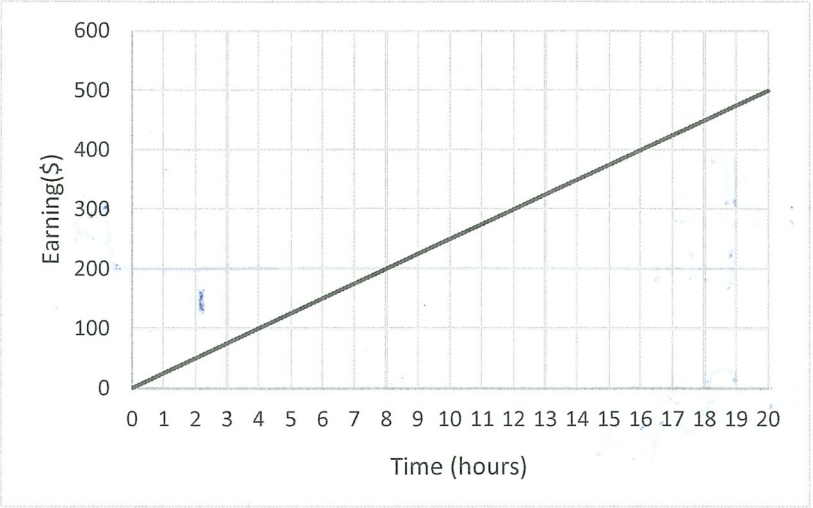
Slope = $\frac{-4}{8} = -\frac{1}{2}$ or -0.5

$y = -\frac{1}{2}x + 10$

Units

When writing the equation of a graph which describes a real world situation we usually use variable names that help us understand what the variables mean, not always x and y.

We also must use units with the slope and y-intercept.



Consider the graph shown. The independent variable is time and the dependant variable is earnings.

Determine an equation for this relation:

$y = 25x + 0$

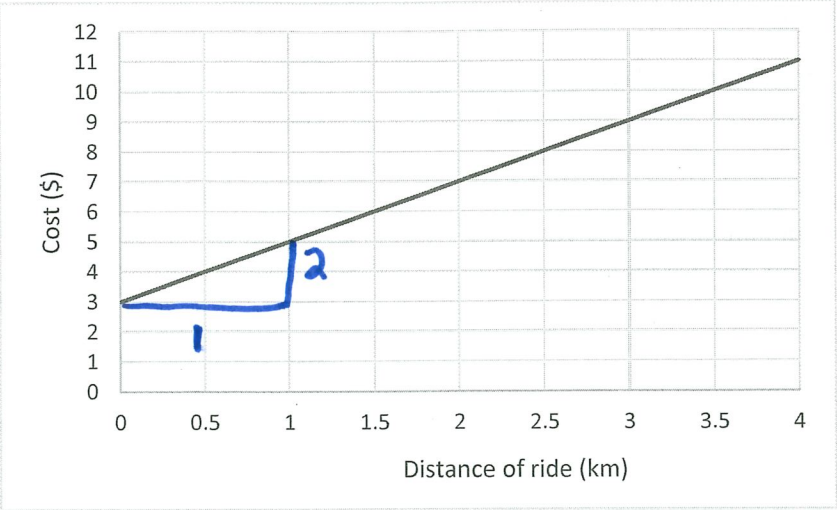
$E = \left(25 \frac{\$}{hr}\right)(t)$

Use the equation to determine how much someone would earn if they worked 3.2 hours.

$E = 25 \frac{\$}{hr} \times 3.2 hr$

$= \$80$

EXAMPLE: Determine an equation with units for the cost of a taxi ride given by the graph shown.



$m = \frac{\$2}{1\,km}$

$= 2\$/km$

$b = \$3$

$C = (2\$/km)(d) + \$3$

Use your equation to determine the cost of a 9.2 km ride.

$C = \frac{2\$}{km}\ 9.2\,km + \$3$

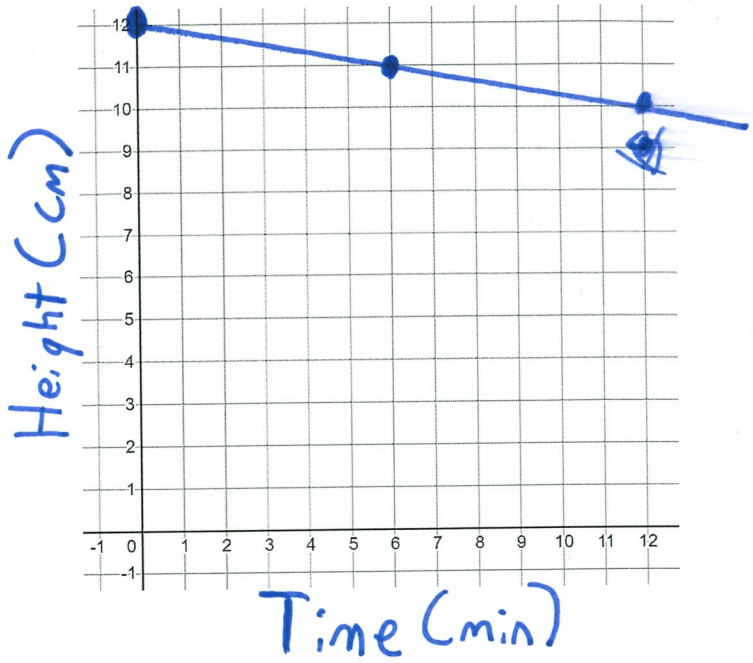
$= \$21$

Often we gather data and have to plot it on a graph to analyze it.

Using the table below:

- label both axis (time is independent)
- plot the data points
- determine the equation for the relation

Height of candle (cm)	Time burning (min)
12	0
11	6.0
10	12



$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-1\,\text{cm}}{6\,\text{min}}$

$h = \left(-\frac{1}{6}\frac{\text{cm}}{\text{min}}\right)(t) + 12\,\text{cm}$ or $h = \left(-0.17\frac{\text{cm}}{\text{min}}\right)(t) + 12\text{c}$

Use your equation to determine the height of the candle after 15 minutes.

$h = -\frac{1}{6}\frac{\text{cm}}{\text{min}} \times 15\,\text{min} + 12\,\text{cm} = 9.5\,\text{cm}$

Rearrange your equation so that time is the subject, then use that equation to determine how long it will take until the candle has burnt out.

$h = \left(-\frac{1}{6}\frac{\text{cm}}{\text{min}}\right)t + 12\,\text{cm}$

$-12\,\text{cm}$

$h - 12\,\text{cm} = \left(-\frac{1}{6}\frac{\text{cm}}{\text{min}}\right)t$

$-6\,\text{min}\,(h - 12\,\text{cm}) = t \rightarrow \frac{-6\,\text{min}}{\text{cm}}(0\,\text{cm} - 12\,\text{cm}) = t$

$72\,\text{min} = t$